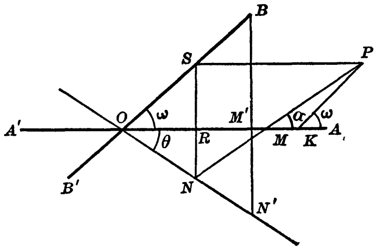

(1) $y = KP = MP \sin \alpha / \sin \omega = b \sin \alpha.$

Also $NM = N'M' = a - b \sin \omega$, and from the triangle ONM, $OM = NM (\sin \alpha \cot \theta + \cos \alpha)$. From triangle $ON'M'$, $\cot \theta = OM'/N'M' = b \cos \omega / NM \therefore OM = b \sin \alpha \cos \omega + NM \cos \alpha = a \cos \alpha - b \sin (\omega - \alpha)$. From triangle MKP, $MK = MP \sin (\omega - \alpha)/\sin \omega = b \sin (\omega - \alpha)$. Hence,

(2) $x = OM + MK = a \cos \alpha.$

Thus (1) and (2) are the desired (parametric) equations of the locus of P.

II. Remarks by the Proposer.

The part of the above proof following (1) can be shortened somewhat thus: Draw the straight line NRS perpendicular to OA, and join P to S. Then $NM/NP = N'M'/N'B = NR/NS$ and hence RM is parallel to SP and also $x = OK = SP$. It now follows that $x = PN \cos \alpha = a \cos \alpha$.

The problem may also be treated geometrically as is done in Rouché and Comberousse, *Traité de Géométrie*, deuxième partie, 8e éd., 1912, pp. 341–345.

332 (Mechanics) [October, 1916]. Proposed by E. E. MOOTS, University of Arizona.

A correct wording of this problem is given in 490 (Geometry) [May, 1916], a solution of which, by A. M. Harding, was published in February, 1917.

198 (Number Theory) [November, 1913; June, 1919]. Proposed by the late ARTEMAS MARTIN.

Prove that every even number is the sum of two prime numbers.

Note by R. C. Archibald, Brown University.

This is Goldbach's empirical theorem and the conjecture appears in a letter to Euler dated June 7, 1742 (*Corresp. Math. Phys.*, ed. Fuss, Vol. 1, 1843, p. 127). The first published statement of the theorem was by E. Waring in his *Meditationes Algebraicæ*, 1770, p. 217. E. Haussner verified the law for numbers up to 10000 (*Jahresbericht der Deutschen Math. Verein.*, Vol. 5, 1896, 62–66), and E. Maillet proved that every even number $\leqq 350000$ (or 10^6 or $9 \cdot 10^6$) is, in default by at most 6 (or 8 or 14), the sum of two primes (*L'Intermédiaire des mathématiciens*, volume 12, 1905, p. 108). These notes are taken from L. E. Dickson's *History of the Theory of Numbers*, volume 1, 1918, where the complete history of the theorem may be found on pages 421–425. No proof of this theorem has yet been discovered.

201 (Number Theory) [December, 1913; June, 1919]. Proposed by E. T. BELL, University of Washington.

Eisenstein proposed (*Crelle*, t. 27, p. 282) as the simplest of several problems: "In the expansion of

$$\frac{1 + z + z^2 + \cdots + z^{p-1}}{(1 - z)^{p-1}} - 1,$$

where p is prime, to show that the coefficients of the various powers of z are all divisible by p."

Solution by R. C. Archibald, Brown University.

$$\frac{1 + z + z^2 + \cdots + z^{p-1}}{(1 - z)^{p-1}} - 1 = \frac{1 - z^p}{(1 - z)^p} - 1,$$

$$= \left(pz - \frac{p(p-1)}{1 \cdot 2} z^2 + \cdots + \frac{p(p-1)}{1 \cdot 2} z^{p-2} - pz^{p-1}\right)(1 - z)^{-p}.$$

Multiplying the first factor of this product by the second factor, expanded, we have the desired result. For, in each factor the coefficients are integers, and p is contained in every coefficient of the first factor.

Also solved by P. J. da Cunha, A. Pelletier, and Elijah Swift.